A discourse to the memory of Gennadi Henkin

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Dear Colleagues, Dear Friends,

Today we are very moved, to be here, altogether, in this place charged with memories, to honor Gennadi Henkin who has left us, almost two years ago.

I would like to thank Andrei Iordan and Tien Dinh who have organized this important meeting and all the participants in this International Colloquium.

I had already the opportunity to highligth the great quality of Gennadi Henkin's works and the variety of the topics covered by him, in October 2012, in the Analysis and Geometry Conference in honour of Gennadi Henkin to congratulate him on winning Stefan Bergman Award.

Born in Moscow on the 26th of October 1942, Gennadi Henkin died in Paris on the 19th of January 2016. He studied in the University of Moscow and, in 1973, he defended his doctoral thesis in Sciences, equivalent to French HDR (Habilitation à diriger des recherches). He had a researcher position at the Institute of Mathematical Economics in the Science Academy of Moscow, from 1967 to 1991.

I discovered G. Henkin's mathematical work in the seventies, a long time before I met him, when I was an assistant professor at the University of Nice. At that time, there were no individual computers nor web. Mathematical papers were typed by secretaries using typewriters and sent to colleagues by the Post.

In Analytic Geometry and in Several Complex Variables, following H. Cartan's famous theory, sheaves theory, homological methods and geometric constructions like analytic covers and blowing up were extremely successful and were the most usual and popular methods in that field of research, particularly in France. Lars Hörmander's L^2 methods coming from the Theory of Partial Differential Equations [Hör1966], were not well known and appeared as a little bit strange and artificial methods in the domain of

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Complex Analysis. The best we could hope using L^2-methods in a bounded pseudoconvex domain Ω in $\mathbb{C}^n$ was to construct global non trivial holomorphic functions in $L^2(\Omega)$. Mathematicians were very far from thinking they could develop a fine theory of holomorphic functions in several variables as the theory of Hardy spaces $H^p(D)$ in one variable [Hoff1962], even if some mathematicians as Léon Ehrenpreis, André Martineau [Mar1967] and François Norguet [And1966] had already obtained successful results with integral representations in several complex variables and would like to extend this theory. Therefore when I learnt in 1971 that a Russian mathematician Gennadi Henkin (not well known at that time) had obtained global L^∞ estimates for the $\bar{\partial}$-equation in a strictly pseudoconvex bounded domain in $\mathbb{C}^n$ [Henk1970] (a difficult problem even in the case of the Ball), it appeared as an amazing and fascinating event, like a thunderclap out of the clear sky.

Moreover, at that time, it was very difficult to interchange results with Russian mathematicians. The first version of the Russian papers were written in Russian language with Cyrillic characters and sometimes an English translation was not easily available.

At that time, Gennadi Henkin powerfully extended the Cauchy-Leray formula of integral representation of holomorphic functions in a stritly convex domain of $\mathbb{C}^n$ to arbitrary differential forms in a strictly pseudoconvex domain. Then he mainly obtained effective, explicite and very precise integral representations solving $\overline{\partial}$-equation on a strictly pseudo-convex domain of $\mathbb{C}^n$ with infinite uniform estimations and a quite simple kernel, directly (closely) related to the geometry of the domain.

Ingo Lieb and Hans Grauert [Lie1970] had obtained the same result using an Enrique Ramirez de Arellano's formula. But G. Henkin's construction was extremely transparent and it had a major impact on the mathematicians dealing with complex analysis in several variables. For it opened the way to studying a lot of problems which seemed to be quite unreachable before (G. Henkin's decisive and founding work). (At first it provides L^p and L^∞ estimates up to the boundary for the solutions of the $\overline{\partial}$ operator.) It particularly opened the way to studying the algebra of bounded holomorphic functions and Hardy's spaces in several complex variables on pseudoconvex domains in $\mathbb{C}^n$.

After this first pionneer work, one of the most striking results in this field of research has been the characterization of the zeros of Nevanlinna class functions with Blaschke condition we have independently obtained G. Henkin [Henk1975] and I ([Sko1975], [Sko1976]). Reading G. Henkin's writing I was pleased to see we had operated with perfectly parallel processes and shown the obvious efficiency in this type of problems of the concept of closed positive current due to Pierre Lelong.

We followed the method Pierre Lelong had successfully used in $\mathbb{C}^n$ as early as 1964 ([Le1964]). We built a holomorphic function F of minimal growth vanishing on a given zeros set X solving in Ω the so called today Lelong-

Poincaré equation connecting F to the current of integration on the given hypersurface X and we use the preceding G. Henkin's integral representation for the solution of the $\overline{\partial}$-equation.

These last results had a major impact especially in France where they have contributed to the development of schools of complex analysis in Bordeaux with Eric Amar and Philippe Charpentier, in Toulouse with Anne Cumenge, in Lille with Anne-Marie Chollet, in Paris with Paul Malliavin and his scholars, with Pierre Dolbeault and Christine Laurent, with Nessim Sibony and Nicolas Varopoulos, with François Norguet and Guy Roos. These results have strongly impacted too Swedish school around Bo Berndtsson and the American school with for instance W. Rudin, S. Krantz, R.M. Range, Y.T. Siu.

On the other hand, using these new explicite integral kernels solving the $\overline{\partial}$-equation, G. Henkin, J. Leiterer and other mathematicians ([Henk1984], [Henk1988]) were able to highlight a complete new approach of the Andreotti-Grauert theory of Stein manifolds and q-convex manifolds and to get more precise results which were out of range before.

In the same vein, using constructions or integral representations as explicite as possible, he has made famous and fundamental contributions to Integral Geometry, Algebraic Geometry and Mathematical Physics through the study of Abel transform, Radon transform and Penrose transform. He has also deeply pushed forward the understanding of the equations of Mathematical Physics more specifically those of inverse problems.

Between 1970 and 1985, G. Henkin was living and working in Moscow and then it was very difficult for a Russian mathematician to go out and visit foreign countries. It's only in June 1987 that he could participate in a colloquium in Montpellier. He could stop in Paris and I met him for the first time. I remember with some emotion his first talk in Paris and his joy when he discovered a part of Paris and the Seine with my family, on a sightseeing boat. It was the beginning of a long friendship.

Then, what seemed to me quite impossible happened in 1989 : the fall of the Berlin wall with all its consequences for the Eastern European countries. G. Henkin could come back several times and quite longer to France. At the end he could apply to a position in Paris 6 University where he was elected as a full professor in 1991. He immediately collaborated to the Complex Analysis Seminar founded by P. Lelong in the sixties and got involved in its administration. G. Henkin played there a leading role and developed new subjects, concerning particularly Abel's transformation, Integral Geometry and Mathematical Physics.

For about twenty years he was supervising so many doctoral theses. He did a magnificent job not only thanks to his high level in mathematics but also because he was able to find new research topics (themes) accessible to our students and was good feeling towards his students and his colleagues. Many of his former students became full professors, associate professors or assistant professors. I shall mention Tien-Cuong Dinh who joined us in October 2005, as a full professor, Stéphanie Nivoche, one of his first students who is now full professor in the University of Nice, Pascal Dingoyan who is associate professor at Paris 6 since 1998 and Luc Pirio who, in 2007, was the first of our team to get a position in the National Center of Scientific Research affiliated to Rennes. Vincent Michel, associate professor in Paris 6 University, has worked a lot with G. Henkin on topics closely related to Mathematical Physics.

For many years G. Henkin was assuming too responsibility for pre-doctoral teaching in pure mathematics which is a key position for defending mathematics in Paris 6 University.

In 1983, he was invited for giving a talk at the World Mathematics Congress in Warsaw.

He was honoured in two international conferences, in Paris, (the first in June 2007, the second one in October 2012).

Gennadi Henkin was above all a scientist, one of the most eminent representatives of Complex Analysis in the world. He was also a simple, self-effacing, always smiling man in spite of a serious illness he battled a long time with great courage. He will continue to be for us a model of scientific engagement and we will always keep in mind the memory of his deep and fruitful works and of all the vocations he has raised.
Gennadi Henkin était avant tout un mathématicien, totalement dévoué à la cause de la science. Il fut l'un des plus éminents représentants de l'analyse complexe à travers le monde. Il était également un homme d'une très grande simplicité, très discret, toujours souriant en dépit d'une grave maladie qu'il combattit jusqu'à la fin, avec beaucoup de courage.
Il restera pour nous un modèle d'engagement au service de la science et nous garderons toujours en nous-mêmes, la mémoire de ses travaux si profonds et si féconds ainsi que celle de toutes les vocations qu'il a fait naître.

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